



Nix, AR., & McGeehan, JP. (1994). High speed bit error rate estimation in frequency selective time-varying channels. In *International Conference on Communications Systems, Singapore* (Vol. 2, pp. 677 - 681). Institute of Electrical and Electronics Engineers (IEEE). <https://doi.org/10.1109/ICCS.1994.474143>

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Link to published version (if available):  
[10.1109/ICCS.1994.474143](https://doi.org/10.1109/ICCS.1994.474143)

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# High Speed Bit Error Rate Estimation in Frequency Selective Time-Varying Channels

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## Abstract

*Over the years many wideband propagation modelling techniques have been proposed for indoor and microcellular environments. However, obtaining system quality of service information is still a time consuming process that often requires extensive computer simulation. In this paper a unique technique is presented that rapidly estimates the average and instantaneous bit error rate for frequency selective time-varying channels. The method was developed to process the wideband channel impulse response information generated by the latest ray-tracing propagation models. The technique combines the speed associated with a theoretical investigation with the flexibility of a conventional simulation. To confirm the accuracy of our algorithm, both the instantaneous and average bit error rates for coherent QPSK are estimated and compared with results obtained by previous authors.*

## I - INTRODUCTION

Digital transmission is particularly sensitive to the wideband nature of the mobile channel. If the rms delay spread becomes an appreciable fraction of the bit rate then intersymbol interference (ISI) results in the generation of an irreducible error floor [1]. This phenomenon results in systems that are time dispersion limited rather than power or interference limited. For dispersion limited channels the maximum bit rate becomes restricted unless complex techniques such as adaptive equalisation are implemented at the receiver. As the transmission rate increases, the time dispersion in the channel becomes a more appreciable fraction of the symbol period. The degree of dispersion in a channel is often characterised by the normalised rms delay spread, obtained by multiplying the rms delay spread of the channel by the system bit rate. If the normalised delay spread is much less than 0.1 the system can be considered narrowband and the majority of bit errors will be caused by interference and/or noise. However, as the normalised delay spread increases beyond 0.01 the system will start to experience wideband characteristics.

For a narrowband system the optimum timing point remains invariant to the effects of fast fading. However, as the normalised delay spread increases, one of the first system effects is an increase in the variation of the optimum timing point. In practice the optimum eye opening is often synchronised to a certain point in the power delay profile. For narrowband channels, the span of the delay profile is small compared to the bit period and hence no noticeable timing effects are observed. However, when the delay profile spans an appreciable

fraction of the bit period it becomes possible for the timing point to vary significantly in sympathy with phase of the received multipaths. Systems such as DECT and CT2 tend to break down in wideband channels initially as a result of their timing loops failing to track these instantaneous delay spread variations. Worst case timing variations occur during deep signal fades where rapid timing flips become possible. Due to the correlation of timing error with signal fade depth, techniques such as antenna diversity are often applied to systems such as DECT to improve their overall quality of service. Unfortunately, as the degree of normalised delay spread increases, even with optimum timing recovery the received eye-diagram tends to close during deep fades. This results in a bursty error pattern with bursts highly correlated with fade depth. It should be remembered that these bursts are irreducible and cannot be lowered with increased transmit power. This burst error phenomenon is extremely important since knowledge of the average bit error rate can be misleading if the system is designed to tolerate random error statistics. For example, in extreme environments (rms delay spreads greater than 500ns) the performance of CT2 degrades badly even though the normalised rms delay spread is just 0.036. Even though the average bit error rate in such locations is around 1 in 10,000, errors do not occur randomly and when a burst does occur an entire data frame can be corrupted and system synchronisation lost. The above illustrates the importance of instantaneous or burst error rate statistics in the prediction of quality of service.

In this paper a technique is described that allows average and burst error rate statistics to be determined using site-specific ray-traced propagation data. The technique uses a hybrid approach, part based on simulation and part based on theory. A full theoretical analysis has not been conducted due to the difficulty in constructing an expression for the calculation of instantaneous or burst bit error rate. To date, all theoretical wideband analysis has concentrated on the prediction of average bit error rates, this work is normally further restricted to worst case non-line-of-sight environments. Due to these theoretical restrictions, the required error function in this study has been obtained empirically using system simulation.

The level of the irreducible error floor is not only dependent of the degree of rms delay spread but also on the bandlimiting filters used in the transceiver. Traditionally, two techniques have been used to determine wideband system performance. The first relies on a mathematical representation of the modem and channel

and, while initially complex to derive, allows rapid generation of results. The second approach uses Monte-Carlo simulation techniques to transmit each data symbol through an appropriate fading channel. While this latter strategy allows a more in-depth analysis of the fading channel, profile shape and timing sensitivity, its drawback lies in the extensive computer run-times required for results.

In this paper a technique is presented that combines the advantages of both theoretical and simulated analysis. The method relies on the continuous extraction of signal and ISI power from the complex convolution of the channel with the combined response of the bandlimiting filters. In this analysis, QPSK is used to illustrate the performance of the algorithm and results are compared with those created using more conventional techniques.

## II - PRINCIPLE OF ESTIMATION

For a narrowband channel, the received eye diagram is distorted by ISI introduced in the bandlimiting filters and noise in the receiver front-end. Providing Nyquist filtering is used in the modem, ISI can be eliminated and error will arise purely due to noise. In practice, root-raised cosine filters are often used in both transmitter and receiver to meet the above criteria. For narrowband scenarios, errors will occur only when the received eye diagram closes due to cases of extreme noise. In general, the distance between the eye openings can be used as a measure of overall system performance. If ISI is introduced by a wideband channel, the resulting eye diagram grows narrower due to the distortion arising from neighbouring symbols. This situation is illustrated in figure 1 where  $Z_{\max}$  represents an upper bound for the received level of ISI. For BPSK transmission,  $Z_{\max}$  can be calculated using equation 1, where  $r(t)$  represents the received waveform and  $kT$ , the  $k$ -th sampling instant. The modulus of  $r(t)$  is assumed to ensure an upper bound for the summation of these ISI terms.

$$Z_{\max} = \sum_{k \neq 0} |r(t - kT)| \quad (1)$$

The error performance of the modem will now be directly related to the value of  $Z_{\max}$  [2]. For a binary system, if the magnitude of  $Z_{\max}$  is less than the magnitude of the wanted sample then the received eye-diagram

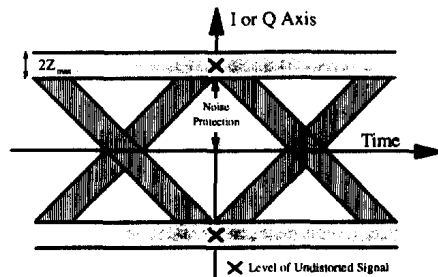


Figure 1: Eye Diagram with Distortion

will narrow but remain open. In this scenario the  $E_b/N_0$  performance of the modem is degraded but no irreducible error rate will occur. Irreducible errors occur whenever the worst case ISI term,  $Z_{\max}$ , becomes greater than the magnitude of the wanted sample. In this paper,  $Z_{\max}$  represents the instantaneous signal to interference ratio and is defined as below:

$$\frac{S}{I} = 20 \log \left( \frac{r(t)}{Z_{\max}} \right) \quad (2)$$

To allow high speed bit error rates to be determined, a further function is required that allows BER to be calculated from  $E_b/N_0$  and instantaneous  $S/I$ . While numerous mathematical expressions exist for this type of function, all are based on assumptions for the interfering signal. One of the most well-developed set of equations makes the assumption that the interfering signal can be represented as a summation of a number of sine waves [2]. While this approach predicts a degrading noise performance as a function of  $S/I$ , at low values of  $S/I$  the sine-wave assumption becomes invalid and the resulting irreducible error floors become erroneous. Since the irreducible errors are of great interest in system design, in this paper such analytical treatments have been discarded in favour of empirically generated look-up tables.

## III - INSTANTANEOUS DISPERSION

For a frequency selective fading environment, both the received  $E_b/N_0$  and the degree of ISI continually fluctuate as a function of temporal variations in the radio channel. Typically, the time-varying complex channel impulse response,  $h(t, \tau)$ , is modelled as below:

$$h(t, \tau) = \sum_{n=1}^L A_n(t) \delta[t - \tau_n(t)] \exp[j\theta_n(t)] \quad (3)$$

where  $A_n(t)$ ,  $\tau_n(t)$  and  $\theta_n(t)$  represent the amplitude, time delay and phase shift of the  $n$ -th arriving path. For simulation purposes a quasi-static channel model is often assumed in which the amplitudes, time delays and arrival angles are assumed constant with only the phase terms being computed as a function of time. In this analysis the phase term for each ray is computed using  $\theta_n(t) = (2\pi v t \cos[\alpha - \beta_n(t)]/\lambda)$ , where  $v$  and  $\alpha$  represent the speed and direction of the mobile receiver and  $\beta_n(t)$  denotes the arrival angle of the  $n$ -th ray. The amplitude, time delay and arrival angle of each multipath were computed using analytical ray-tracing techniques [1].

If  $d(t)$  is used to represent the impulse response of the desired data, after bandlimiting and matched filtering, the resulting waveform,  $x(t)$ , can be computed from  $d(t) * p(t)$ , where  $*$  denotes the process of convolution and  $p(t)$  represents the combined impulse response of the transceiver filters. The baseband representation of the

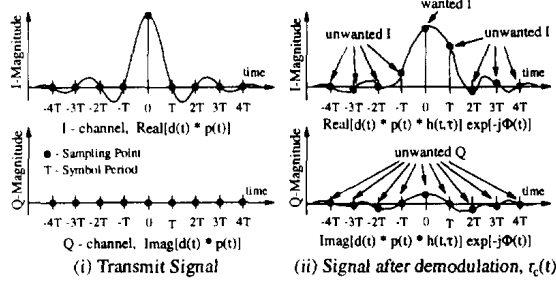


Figure 2: Calculating Wanted and Unwanted Samples

received signal waveform,  $r(t)$ , can now be obtained by convolving  $x(t)$  with the time varying impulse response of the mobile channel,  $h(t, \tau)$ . If coherent detection is assumed, it is also necessary to cancel the phase variations,  $\Phi(t)$ , introduced by the mobile channel. As shown in equation 4, this is achieved by multiplying the received waveform by  $\exp[-j\Phi(t)]$ , where  $\Phi(t)$  is obtained from the vector summation of the multipath signals.

$$\begin{aligned} r_c(t) &= r(t) \exp[-j\Phi(t)] = [x(t) * h(t, \tau)] \exp[-j\Phi(t)] \\ &= [d(t) * p(t) * h(t, \tau)] \exp[-j\Phi(t)] \end{aligned} \quad (4)$$

To mathematically evaluate the degree of time-varying ISI, a data sequence is required to allow the ISI components to be calculated. Figure 2(i) shows the sequence used in this paper, to sound the channel a single filtered impulse is sent in the I-domain. Ideally, for a narrow-band channel there is no I-Q cross distortion and symbol energy in the I-domain is unaffected by that in the Q-domain. In a frequency selective channel, the recovered phase angle,  $\Phi(t)$ , is no longer correlated with the entire signal bandwidth. As a result, the I and Q impulse streams not only spread in out in time, but also suffer from I-Q cross distortion. For a wideband channel, Figure 2(ii) shows the type of distorted waveform expected at the receiver. The original filtered response now suffers from severe ISI since the transmitted data symbol is no longer confined to a single timing instance. I-Q cross-distortion also occurs in figure 2(ii) and this results in energy 'spilling' into the quadrature component. Assuming these ISI terms add-up constructively, an upper bound for the wanted ( $w$ ) and the unwanted ( $z$ ) signals can be computed as below.

$$w = |r_I(t + \tau_a)| \quad (5)$$

wanted I

$$z = \sum_{k \neq 0} |r_I(t + \tau_a + kT_s)| + \sum_k |r_Q(t + \tau_a + kT_s)| \quad (6)$$

unwanted I                      unwanted Q

In this particular analysis timing is assumed to be fixed at the centroid of the power delay profile,  $\tau_a$  [2]. However, it should be remembered that system performance could be further improved by dynamically tracking the optimum received sampling instant. However, such tracking is only appropriate for channels where the data rate is high compared to the fade rate [3]. For outdoor vehicular based systems, the fade rate is often too high to allow effective tracking and in such cases timing is then fixed at an estimate of the average delay.

The instantaneous signal to interference ( $S/I$ ) ratio can now be calculated as  $20\log(w/z)$ . The received signal power, and hence the  $E_b/N_0$ , is computed from the vector summation of the arriving rays. For a given modulation scheme, BER can now be derived from these values of  $E_b/N_0$  and  $S/I$ . Unfortunately, an accurate mathematical expression for this burst error rate function has not been derived and would be complex to develop, in this paper the relationship is constructed using a look-up table with entries computed using software simulation. For coherent QPSK, many static simulations were performed at various data rates and delay spreads to generate instantaneous or burst error rate statistics as a function of  $E_b/N_0$  and  $S/I$ . Figure 3 illustrates the static AWGN results for coherent QPSK with varying degrees of signal to ISI power. As expected, as the degree of ISI increases, the calculated value of  $S/I$  decreases and more energy per bit will be required to maintain a given error rate. For QPSK, when the  $S/I$  ratio falls below 0 dB an irreducible error floor appears and the error rate cannot be lowered further with increasing  $E_b/N_0$ . To allow instantaneous BER to be calculated from any value of  $E_b/N_0$  and  $S/I$ , a look-up table,  $f(E_b/N_0, S/I)$ , was created by interpolating the full set of AWGN+ISI simulation results. In practice, due to relative motion between transmitter and receiver, both  $E_b/N_0$  and  $S/I$  become time varying parameters.

$$BER(t) = f\left(\frac{E_b(t)}{N_0}, \frac{S}{I}(t)\right) \quad (7)$$

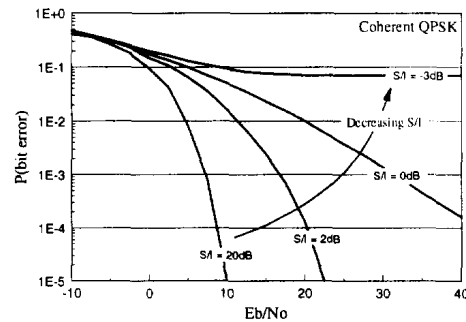


Figure 3: Static AWGN + ISI Performance (QPSK)

$$BER_{ave} = \frac{1}{N} \sum_{m=1}^N BER(mT_s) \quad (8)$$

$$\left(\frac{E_b}{N_0}\right)_{ave} = \frac{1}{N} \sum_{m=1}^N \frac{E_b}{N_0}(mT_s) \quad (9)$$

To obtain the average BER, the complex channel impulse response is processed to compute time varying waveforms for  $E_b/N_0$  and  $S/I$  - see figures 4(i) and 4(ii). For any time  $t$ , the look-up table can be used to generate the instantaneous BER for the corresponding values of  $E_b/N_0$  and  $S/I$ . Average BER is then calculated by averaging these instantaneous bit error rates over a suitable number of fades. Equations 7,8 and 9 show the resulting mathematical expressions for instantaneous BER, average BER and average  $E_b/N_0$  for  $N$  iterations at a sampling frequency of  $1/T_s$  Hertz.

#### IV - RESULTS

Figure 4 shows the graphical output from a typical high-speed simulation run. To produce these results  $p(t)$  was assumed to have a standard raised cosine impulse response. Figures 4(i) and 4(ii) show the received narrowband signal power and  $S/I$  ratio as a function of time over approximately 40 fades. From knowledge of the receiver's noise floor,  $E_b/N_0$  can be generated and, together with the corresponding  $S/I$  ratio, an estimate of the instantaneous or burst BER extracted from the look-up table defined in section III. Figure 4(iii) shows the resulting irreducible error pattern assuming the transmission of 8,192 data bits (the bit error rate function defined in equation 7 was used to determine which of the 8,192 data bits were statistically received in error). To confirm the validity of these results, figure 4(iv) shows the error pattern arising from a back-to-back bit level simulation. Errors occur whenever the  $S/I$  ratio falls significantly below 0 dB, interestingly these locations are also well correlated with narrowband fades [1-3]. However, since only 8,192 bits were used in the simulation, it is possible that there are insufficient bits during a fade for some of the burst errors to be displayed. Indeed, this phenomenon is responsible for the slight differences that exist between the estimated and simulated results.

It appears that an irreducible error burst is not determined by fade depth but by the instantaneous value of  $S/I$ . For many line-of-sight channel profiles a peak in the received impulse response lowers the degree of average dispersion (see figure 5). However, during a fade it is possible for the phase of the rays to align such that the peak of the profile is destructively cancelled - see figure 5(ii). The resulting 'flat' impulse response now exhibits a large degree of *instantaneous* dispersion. Alternatively, the entire profile can fade uniformly as shown in figure 5(i). If this condition occurs then the value of dispersion will be low during the fade. This argument explains the observed burst errors in systems such as DECT where *all* irreducible error bursts occur in deep

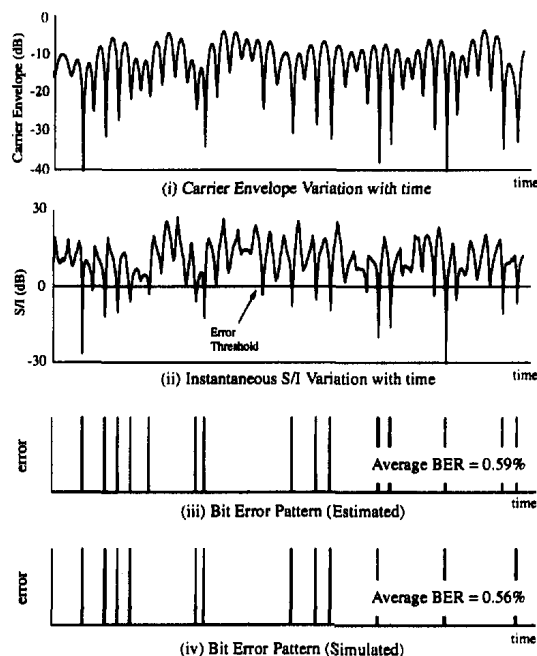


Figure 4: QPSK Waveforms ( $d=0.2$ , roll-off=0.5)

fades but not *all* deep fades produce error bursts [4]. In addition, since rms delay spread is calculated without the use of any phase information, the parameter remains constant during fades. This limitation prevents the rms delay spread from predicting the bursty nature of the received errors. In practise, rms delay spread can be used as an indicator of average bit error but it should be remembered that true instantaneous and average bit error rate will be a function of the phase of each ray, the shape of the profile, the deterministic nature of the channel and the channel filtering used in the transceivers.

Figure 6 shows the resulting average BER for a frequency selective Rayleigh fading channel. In this example, four values of normalised delay spread were assumed with the irreducible error floor rising with increasing dispersion. These results were generated us-

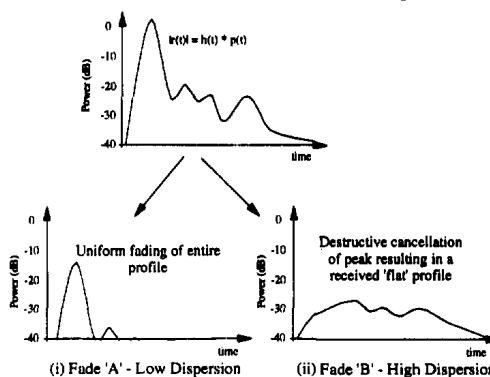


Figure 5: Instantaneous Dispersion vs Fade Depth

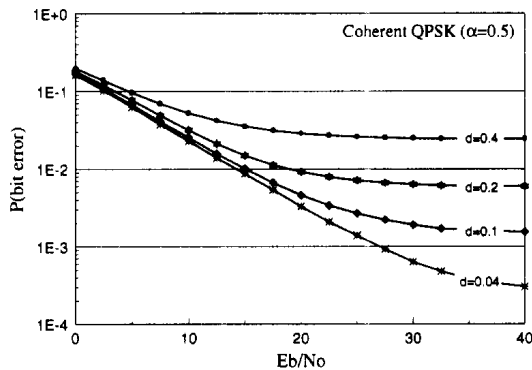


Figure 6: AWGN Frequency Selective QPSK

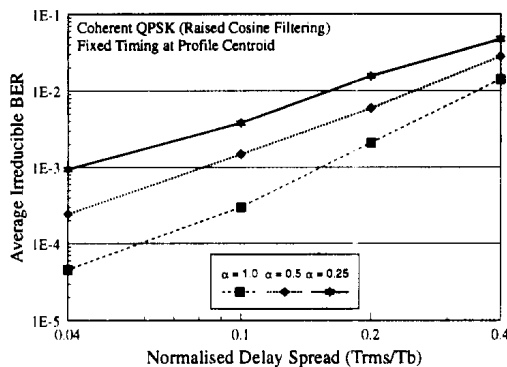


Figure 7: Irreducible BER vs Filter Roll-off (QPSK)

ing the algorithms described in this paper. A raised cosine filter with a roll-off of 0.5 was also assumed.

Unlike our previous estimation technique [1], the above approach dynamically calculates  $S/I$  based on the instantaneous channel conditions and the type of bandlimiting filters being used. Hence, BER can now be produced using a single look-up table for any combination of transmit and receive filter. To illustrate this point, figure 7 shows the irreducible error floor in a frequency selective Rayleigh fading channel at three different values of raised cosine roll-off factor. Each result was generated by averaging over a distance of 40 wavelengths with approximately 1,000 samples per fade (if results are required in the order of seconds then reasonable approximations can be still be achieved using 32 samples per fade). These results are in excellent agreement with those produced in [3] and show how the irreducible error floor rises as the filter roll-off factor is tightened. Assuming a bit error threshold of 1 in 1000 and an indoor rms delay spread of 100ns, figure 7 indicates that data rates are restricted to 400kb/s, 800kb/s and 1.4Mb/s for roll-off factors of 0.25, 0.5 and 1.0 respectively. For a typical outdoor rms delay spread of 2 $\mu$ s, data rates are restricted to 20kb/s, 40kb/s and 70kb/s.

## IV - CONCLUSIONS

This paper has presented a new technique for rapidly calculating average and burst BER in a realistic mobile fading environment. Since the calculation is based on an analysis of the complex channel impulse response, BER can be computed for any shape of delay profile and any value of K-factor. This type of approach allows system factors such as timing loops and bandlimiting filters to be incorporated into the study. The method can also be applied to other types of modulation scheme by the generation of an appropriate look-up table. Combining the techniques described in this paper with a suitable ray-tracing propagation model, rapid area wide quality of service predictions become possible.

## ACKNOWLEDGEMENTS

The authors would like to acknowledge the mathematical assistance of Mr Y.C. Chow in the preparation of this paper. The authors are also grateful to the Centre for Communications Research for the provision of bench and computing facilities.

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